# MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

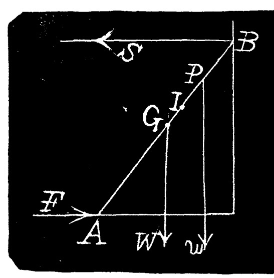
## SOLUTIONS TO PROBLEMS.

**9. Proposed by CHARLES E. MYERS, Canton, Ohio.**

A ladder inclined at an angle of 60° to the horizon rests with one end on a *rough* pavement and the other against a *smooth* vertical wall. If the coefficient of friction between the foot of the ladder and the pavement is $\frac{1}{6}\sqrt{3}$, to what height can a man ascend before the ladder will begin to slip?

**Solution by B. F. FINKEL, Professor of Mathematics, Physics, and Astronomy, Kidder Institute, Kidder, Missouri.**

Let $AB$ be the ladder, length $2l$; $\theta$ its inclination to the horizon, $W$ its weight and $G$ its center of gravity; $AP=x$, the distance the man ascends and $w$ his weight; $I$, the center of gravity of the man and ladder; $F=\mu$, the coefficient of friction; and $S$ and $R$ the normal resistance at $A$ and $B$, respectively.




Then we have $S-F=0\ldots.(1)$ and, $W+w-R=0\ldots.(2)$. Taking moments about $I$, we have $W.GI=w.PI$, or $W.GI-w.PI=0\ldots.(3)$. But $GI+PI=x-\frac{1}{2}l$. Hence, from (3), $GI=\frac{w}{W+w}(x-\frac{1}{2}l)$. Taking moments about $A$, $(W+w).AI=AB.R(\cos\theta-\mu\sin\theta)$ or $(W+w)\left[\frac{w}{W+w}(x-\frac{1}{2}l)+\frac{1}{2}l\right]\cos\theta=2lR(\cos\theta-\mu\sin\theta)\ldots.(4)$. (2) and (4) give $2l(W+w)(\cos\theta-\mu\sin\theta)=(Wl+wx)\cos\theta$, whence $x=\frac{2l(W+w)(1-\mu\tan\theta)-Wl}{w}$. But $\mu=\frac{1}{6}\sqrt{3}$ and $\tan\theta=\sqrt{3}$.

$\therefore AP=x=l$. Hence the man can ascend to the middle point of the ladder.

[Note.—This problem and its solution was selected for publication in the Aug. No. But it was lost in the office of the publishers and thus we are unable to credit our contributors for their solutions to it. ED.]

**10. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton Virginia.**

A paraboloid floats in a liquid which fills a fixed paraboloid shell; both the paraboloid and the shell have their axis vertical and their vertices downward; the latus rectum of the paraboloid and shell are equal, and the axis of the shell is $m$ times that of the paraboloid. If the paraboloid be pressed down until its vertex reaches the vertex of the shell, so that some of the liquid overflows, and then released, it is found that the paraboloid rises until it is just wholly out of the liquid, and then begins to fall. Prove that (1) the densities of the paraboloid and liquid are in the ratio.

$2[m^2+m+1=(m+1(\sqrt{m^2-1})]:3\sqrt{(m+1)\div(m-1)}$, the free surface of the liquid being supposed to remain horizontal throughout the motion; and (2)

if cone and conical be used, is the ratio $3\ [m^4-1-(m^3-1)\sqrt[3]{(m^3-1)}:4\sqrt[3]{(m^3-1)}$, the vertical angles being equal.

**Solution by G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

In the two positions in which the velocity of the paraboloid is zero, the heights of the centre of gravity of the paraboloid and liquid are equal.

Let $\rho$ be the density of the paraboloid, $\sigma$ of the liquid, $h$ the height of fhe paraboloid.

$y^2=4ax$ the equation to the parabola of revolution which generates the paraboloid; $x$ the height of the surface of the liquid in the second position.

Then $2a\pi x^2=2a\pi m^2h^2-2a\pi h^2=2a\pi(m^2-1)h^2$.

$\therefore\ x=\sqrt{m^2-1}\ h$.

Hence $\frac{2}{3}.mh.2a\pi m^2h^2\sigma-\frac{2}{3}h.2a\pi h^2(\sigma-\rho)=\frac{2}{3}\sqrt{m^2-1}\ h.2a\pi(m^2-1)h^2\sigma$
$+\{\sqrt{m^2-1}+\frac{2}{3}\}h.2a\pi h^2\rho$.

$$\therefore\ \tfrac{2}{3}m^3\sigma-\tfrac{2}{3}(\sigma-\rho)=\tfrac{2}{3}\sqrt{(m^2-1)^3}\sigma+(\sqrt{m^2-1}+\tfrac{2}{3})\rho.$$

$$2\{m^3-1-\sqrt{(m^2-1)^3}\}\sigma=3\sqrt{m^2-1}\ \rho,$$

$$2\{m^2+m+1-(m+1)\sqrt{m^2-1}\}\sigma=3\sqrt{(m+1)\div(m-1)}\rho.$$

$$\therefore\ \rho:\sigma=2\{m^2+m+1-(m+1)\sqrt{m^2-1}\}:3\sqrt{(m+1)\div(m-1)}.$$

For the cone and conical cup, we have $\frac{1}{3}\pi x^2\tan^2\beta=\dfrac{\pi m^3h^3}{3}\tan^2\beta$
$-\dfrac{\pi h^3}{3}\tan^2\beta$ where $\beta$ is the semi-vertical angle of the cone.

$$\therefore\ x=h\sqrt[3]{m^3-1}.$$

Hence $\frac{3}{4}.mh.\dfrac{\pi m^3h^3}{3}\tan^2\beta\sigma-\frac{3}{4}h.\dfrac{\pi h^3}{3}\tan^2\beta(\sigma-\rho)$

$$=\tfrac{3}{4}h\sqrt[3]{(m^3-1)}\ \frac{\pi(m^3-1)h^3}{3}\tan^2\beta\sigma+\{\sqrt[3]{(m^3-1)}+\tfrac{3}{4}\}h.\frac{\pi h^3}{3}\tan^2\beta\rho.$$

$$\therefore\ \tfrac{3}{4}m^4\sigma-\tfrac{3}{4}(\sigma-\rho)=\tfrac{3}{4}(m^3-1)\sqrt[3]{m^3-1}\ \sigma+\{\sqrt[3]{m^3-1}+\tfrac{3}{4}\}\rho.$$

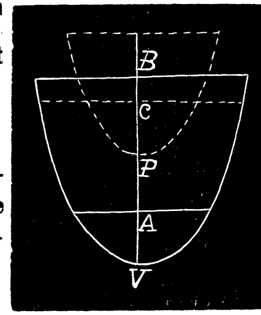
$$\therefore\ 3\{m^4-1-(m^3-1)\sqrt[3]{m^3-1}\}=4\sqrt[3]{m^3-1}\ \rho.$$

$$\therefore\ \rho:\sigma=3\{m^4-1-(m^3-1)\sqrt[3]{m^3-1}\}:4\sqrt[3]{m^3-1}.$$

**II. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.**

Let $VA(=a)$ be the axis of the paraboloid; $VB(=ma)$, that of the shell; $2p$, their common latus rectum; $\rho$, the ratio of the density of the solid to

to that of the liquid; $P$, the vertex of the paraboloid at time $t$, the depth of the liquid being $Vc$; $VP=x$, $Pc=y$.




The paraboloid rises with constant acceleration $\frac{1-\rho}{\rho}g$ and, therefore, its upper surface reaches that of the liquid with velocity $\sqrt{2ga(m-1)\frac{1-\rho}{\rho}}$. The acceleration now becomes variable. As the solid continues to rise the level of the liquid is lowered. The upward pressure on the paraboloid is always proportional to $y^2$. $y$ can be found from the equation

$$\pi p[m^2a^2-(x+y)^2]=\pi p(a^2-y^2).$$

$$y=\frac{m^2a^2-a^2-x^2}{2x}.$$

The equation of motion is $\frac{d^2x}{dt^2}=cy^2-g$, where $c$ is a constant.

Substituting the value of $y$, multiplying by $2dx$, and integrating,

$$\left(\frac{dx}{dt}\right)^2=\frac{c}{2}\left[\frac{x^3}{3}+2a^2(1-m^2)x+\frac{a^4(2m^2-m^4-1)}{x}\right]-2gx+c_1.$$

$c$ can be found from the first differential equation since, when $y=a$,

$$\frac{d^2x}{dt^2}=\frac{1-\rho}{\rho}g.$$

Hence $c=\frac{g}{a^2\rho}$. $c_1$ can be determined from the second differential equation for, when $x=a(m-1)$, $\left(\frac{dx}{dt}\right)^2=2ga(m-1)\frac{1-\rho}{\rho}$.

$$c_1=\frac{4ga}{3\rho}(m-1)(m^2+m+1).$$

Substituting these values of $c$ and $c_1$,

$$\left(\frac{dx}{dt}\right)^2=\frac{g}{2a^2\rho}\left[\frac{x^3}{3}-2a^2(m^2-1)x-\frac{a^4(m^2-1)^2}{x}\right]-2gx+\frac{4ga}{3\rho}(m-1)$$

$(m^2+m+1)$. $\frac{dx}{dt}=0$ at the instant that the paraboloid is wholly without the liquid. At this time $y=0$, and, therefore, $x=a\sqrt{m^2-1}$.

Substituting these values and solving for $\rho$,

$$\rho=\frac{2[(m-1)(m^2+m+1)-(m^2-1)^{\frac{3}{2}}]}{3(m^2-1)^{\frac{1}{2}}}=\frac{2[m^2+m+1-(m+1)\sqrt{m^2-1}]}{3\sqrt{\frac{m+1}{m-1}}}.$$

2nd. The velocity with which the base of the cone reaches the free surface of the liquid is $\sqrt{2ga(m-1)\frac{1-\rho}{\rho}}$, as before. Afterwards the bouyant

force is proportional to $y^3$ and $y$ may be found from $\frac{\pi\tan^2\infty}{3}[m^3a^3-(x+y)^3]$ $=\frac{\pi\tan^2\infty}{3}(a^3-y^3)$ where $\infty$ is the semi-vertical angle of the cone.

$$y=-\frac{x}{2}+\sqrt{\frac{a^3(m^3-1)}{3x}-\frac{x^2}{12}}.$$

The equation of motion is $\frac{d^2x}{dt^2}=cy^3-g$.

Since, when $y=a$, $\frac{d^2x}{dt^2}=\frac{1-\rho}{\rho}g$, $c=\frac{g}{a^3\rho}$.

Substituting this and the value of $y$

$$\frac{d^2x}{dt^2}=\frac{g}{a^3\rho}\left[\left(\frac{a^3(m^3-1)}{3x}+\frac{2x^2}{3}\right)\sqrt{\frac{a^3(m^3-1}{3x}-\frac{x^2}{12}}-\frac{a^3(m^3-1)}{2}\right]-g.$$

Integrating,

$$\left(\frac{dx}{dt}\right)^3=\frac{2g}{a^3\rho}\left[-2x\left(\frac{a^3(m^3-1)}{3x}-\frac{x^2}{12}\right)^{\frac{3}{2}}-\frac{a^3(m^3-1)}{2}x\right]-2gx+c_1.$$

When $x=a(m-1)$, $\left(\frac{dx}{dt}\right)^2=2ga(m-1)\frac{1-\rho}{\rho}$.

$$\therefore\ c_1=\frac{3ag}{2\rho}(m^4-1).$$

When $y=0$ and, consequently, $x=a\sqrt[3]{m^3-1}$, $\frac{dx}{dt}=0$.

$$\therefore\ 0=\frac{2g}{a^3\rho}\left(-\frac{3a^4(m^3-1)^{\frac{4}{3}}}{4}\right)-2ga\sqrt[3]{m^3-1}+\frac{3ag}{2\rho}(m^4-1),\text{ and}$$

$$\rho=\frac{3[m^4-1-(m^3-1)\sqrt[3]{m^3-1}]}{4\sqrt[3]{m^3-1}}.$$

## PROBLEMS.

**16. Proposed by A. H. BELL, Hillsboro, Illinois.**

An iron bar 20 feet long and weighing 2,000 lbs. leans against a wall at angles 30°, 45°, and 80°. How much pressure does the floor and wall receive?

**17. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

Find the law of density of strings collected into a heap at the edge of a table with the end just over the edge, so that equal masses may always pass over in equal times.